Moment methods in extremal geometry Layman's talk

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Extremal geometry



Extremal geometry







Coding theory (Example: Voyager probes)



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- Cryptography



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- Modeling particle systems (Symmetry?)



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Modeling particle systems (Symmetry?)

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Proof:



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Proof:



Second claim: One cannot arrange 13 billiard balls such that all of them kiss a 14th billiard ball

 First claim: One can arrange 12 billiard balls such that all of them kiss a 13th billiard ball

Proof:



- Second claim: One cannot arrange 13 billiard balls such that all of them kiss a 14th billiard ball
- ► Goal: Develop techniques to find proofs for claims like these

My thesis introduces a concept of moments for geometric configurations

Tools

Combine moment formulation with

- optimization,
- harmonic analysis,
- and real algebraic geometry

to build a computer program that generates proofs

Optimization

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 Duality: Each maximization problem has a corresponding minimization problem (and vice versa)

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- Different ways of writing f:

•
$$f(x) = x(x^3 - 10x^2 + 27x - 10) + 1$$

•
$$f(x) = (x^2 - 5x + 1)^2$$

Problem in for instance coding theory

Problem in for instance coding theory \downarrow Problem in extremal geometry

Problem in for instance coding theory \downarrow Problem in extremal geometry \downarrow Moment formulation of the problem

Problem in for instance coding theory \downarrow Problem in extremal geometry \downarrow Moment formulation of the problem \downarrow Use optimization, harmonic analysis, and real algebraic geometry

Thank you!

